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Assumed Places of the Comparison-Stars.

Ref.	Mean R.A. 1907·0.	Reduction to Appar. R.A.	Mean N.P.D. 1907·0.	Reduction to Appar. N.P.D.	Authority.
	h m s	s	° ′ ″	″	
(*a*) (*b*)	4 15 37·21	+0·31	74 17 16·8	+0·8	Küstner, *Bonn Veröff.*, No. 4, p. 97.
(*c*)	6 6 14·32	+0·11	72 36 4·7	+1·8	Berlin, A (*A.G.*) 1973
(*d*, *e*)	6 28 53·94	+0·06	72 42 7·1	+1·8	Berlin, A (*A.G.*) 2231
(*f*)	6 30 27·59	+0·05	72 43 28·2	+1·8	Berlin, A (*A.G.*) 2252
(*g*)	7 8 31·76	+0·02	73 15 38·2	+1·8	Berlin, A (*A.G.*) 2702
(*h*) (*i*)	7 24 8·76	+0·01	73 38 39·6	+1·7	Berlin, A (*A.G.*) 2862
(*j*)	8 16 44·43	−0·05	74 56 8·1	+1·6	Berlin, A (*A.G.*) 3303
(*k*)	9 31 29·11	−0·02	78 44 12·8	+1·2	Leipzig, I (*A.G.*) 3809
(*l*)	9 33 46·34	−0·03	78 48 2·5	+1·1	Leipzig, I (*A.G.*) 3820
(*m*)	10 2 58·18	−0·05	79 32 46·1	+1·0	Leipzig, I (*A.G.*) 3952 (P. M.'s $-0^{s}{\cdot}0082$ and $+0''{\cdot}038$.
(*n*)	9 58 36·74	−0·04	79 39 3·1	+1·0	Leipzig, I (*A.G.*) 3933

In the computation of the parallaxes the adopted value of the Sun's mean horizontal parallax is 8″·80, and the geocentric distances Δ are taken from the *Astronomische Nachrichten*, Nos. 4191, 4194, and 4196.

Radcliffe Observatory, Oxford:
1908 *May* 30.

The Orbit of Jupiter's Eighth Satellite. By P. H. Cowell and A. D. Crommelin.

In this paper the time is measured in units of 16 days from 1908 January 12·0 Paris mean time. The astronomical unit of length is used.

Paris mean time has been used because the Connaissance des Temps gives both heliocentric and geocentric longitudes and latitudes of Jupiter referred to the same equinox, the true equinox of date.

With 16 days as the unit of time,

log mass of Sun = 8·87 943
,, ,, Jupiter = 5·85 934

It will be seen in the later parts of this paper that the Sun's disturbing force varies during the early part of 1908 between 10 and 6 per cent. of the attractive force of Jupiter, and is therefore by no means negligible. Moreover, if the attempt were made to express the co-ordinates of the satellite with the help of Delaunay's algebraical lunar theory, it would be found that Delaunay's quantities *e*, *e*′ and *m* take such large arithmetical values that Delaunay's series are insufficient for the purpose. Hence it is

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best to calculate the orbit by quadratures, discarding the idea of an ellipse.

Let the jovicentric co-ordinates at time $n+t$ referred to three rectangular directions be

$$x_n + a_1t + a_2t^2 + a_3t^3 + a_4t^4 + a_5t^5 + a_6t^6 +$$
$$y_n + b_1t + b_2t^2 + b_3t^3 + b_4t^4 + b_5t^5 + b_6t^6 +$$
$$z_n + c_1t + c_2t^2 + c_3t^3 + c_4t^4 + c_5t^5 + c_6t^6 +$$

then we have $\quad x_{n+1} = 2x_n - x_{n-1} + 2a_2 + 2a_4 + 2a_6.$

Again, given the co-ordinates at any time, the accelerations can be calculated in accordance with Newton's law of gravitation. If X denote the x acceleration at time $n+t$,

$$X = 2a_2 + 6a_3t + 12a_4t^2 + 20a_5t^3 + 30a_6t^4.$$

Hence $\qquad X_n = 2a_2$

and $\qquad \frac{1}{12}(X_{n+1} - 2X_n + X_{n-1}) = 2a_4 + 5a_6 + \ldots$

The left-hand side of this last equation may be described in words as one-twelfth of the second difference of X_n, and it will be denoted for brevity by P_n.

Hence we have $\quad x_{n+1} = x_n + (x_n - x_{n-1}) + X_n + P_n$

with one error of $3a_6$, which we shall assume to be negligible.

Now let us assume that x_{n-2}, x_{n-1}, x_n have been calculated, and X_{n-2}, X_{n-1}, X_n, and therefore also P_{n-2}, P_{n-1}. We cannot at present calculate P_n, for this requires a knowledge of X_{n+1}, and therefore of $x_{n+1}, y_{n+1}, z_{n+1}$; but we may guess at the value of P_n with the help of the values of P_{n-2}, P_{n-1} (or for the first few values of n, where P_{n-2} has not been calculated, by trial and error); we then obtain a provisional value of x_{n+1} (and similarly y_{n+1}, z_{n+1}); hence we calculate X_{n+1}, and so obtain a revised value of P_n; the correction ΔP_n is then added to x_{n+1}, and in practice it is found that $X_{n+1}, Y_{n+1}, Z_{n+1}$ are unaffected.

We give a numerical application of the above method, taken from our calculations of a provisional orbit. In the illustration it is assumed that the orbit has already been calculated as far as $t=5$ (April 1); it is required to extend it to $t=6$ (April 17). For the sake of clearness, quantities already calculated are written without brackets, quantities guessed at are written in round brackets, and fresh quantities calculated accurately are enclosed in square brackets.

$x_5 = +\cdot 0511\ 598$	$y_5 = -\cdot 1691\ 203$	$z_5 = -\cdot 0173\ 081$
$x_5 - x_4 = -\cdot 0160\ 908$	$y_5 - y_4 = -\cdot 0004\ 222$	$z_5 - z_4 = +\cdot 0095\ 069$
$X_5 = -\cdot 0005\ 159$	$Y_5 = +\cdot 0020\ 967$	$z_5 = +\cdot 0002\ 300$
$(P_5) = +\cdot 0000\ 027$	$(Q_5) = +\cdot 0000\ 015$	$(R_5) = -\cdot 0000\ 020$

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Hence by addition,

$$(x_6) = +\cdot 0345\ 558,\ (y_6) = -\cdot 1674\ 443,\ (z_6) = -\cdot 0075\ 732.$$

Hence we calculate X_6, Y_6, Z_6, and hence

$$[P_5] = +\cdot 0000\ 029,\ [Q_5] = +\cdot 000\ 014,\ [R_5] = -\cdot 0000\ 020.$$

Hence applying the small differences between the true and provisional values of P_5, Q_5, R_5,

$$[x_6] = +\cdot 0345\ 560,\ [y_6] = -\cdot 1674\ 444,\ [z_6] = -\cdot 0075\ 732.$$

The foregoing sufficiently explains how to calculate the motion from assumed initial positions; we now deal with the problem how to find an approximate orbit from the observations.

Retaining the previous notation, and interpolating a little among the observations, we shall assume as known the geocentric angular positions at three equidistant times $n-\tau$, n, and $n+\tau$.

Reasoning exactly as before we have

$$x_{n+\tau} - 2x_n + x_{n-\tau} = X_n\tau^2 \text{ with an error of } 2a_4\tau^4.$$

If we take the x direction at right angles to the geocentric directions of the satellite at times $n-\tau$ and $n+\tau$, then our ignorance of the geocentric distances at these times produces no uncertainty in the numerical values of $x_{n+\tau}$ and $x_{n-\tau}$; we therefore have one unknown quantity only occurring, viz. the geocentric distance at time n; and the above equation, by trial and error, will determine this quantity.

The corresponding equations for y and z then determine the geocentric distances on the first and third occasions.

We may then revise our solution so as to correct for the residuals $2a_4\tau^4$, etc.

In practice it is not necessary to take the x axis at right angles to the first and third geocentric distances; we shall then have to deal with three simultaneous equations for three unknown geocentric distances. The idea of resolution perpendicular to the outer geocentric distances is perhaps useful for forming a distinct mental conception of the process.

We shall not go into numerical details in connection with the method just sketched. It may be anticipated, however, that the solution is a dual one, or that two orbits which may be distinguished as direct and retrograde can be found to satisfy three observations. It unexpectedly turned out that the jovicentric distance on February 28 was less on the supposition of a direct orbit than for a retrograde orbit; but a very large eccentricity resulted from the former hypothesis. For this reason combined with the reasons stated in *M.N.*, lxviii. p. 457, the retrograde solution was chosen as the one to be worked up. It will be seen that the corresponding orbit closely follows the observations. Now the dual solution implies that there are two distinct ways of making the coefficients of the squares of the time agree with observation. Of these two solutions the retrograde solution makes the coefficients of the third and fourth

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powers of the time also agree with observation. Of course it is not inconceivable that the direct solution might also give cube and fourth terms agreeing with observation, but it is *a priori* distinctly improbable. We have not, therefore, examined the direct solution (or the question of a possible planetary solution) with any minuteness. However closely a direct solution was found to fit the observations, it would be impossible at the present moment to reject the possibility that the retrograde solution was the right one.

The following table contains full details of our provisional orbit. The axes are drawn as follows :—x towards the first point of Aries for 1908·0, y towards longitude 90°, z towards the pole of the ecliptic

$x\ y\ z$ are the jovicentric co-ordinates of the satellite,
$x'\ y'\ z'$,, ,, ,, Sun
$X_1\ Y_1\ Z_1$ are the accelerations of Jupiter on the satellite
$X_2\ Y_2\ Z_2$,, ,, the Sun ,, ,,
$X_3\ Y_3\ Z_3$,, ,, the Sun on Jupiter reversed
X Y Z are the sums of the three preceding quantities.
P Q R are the twelfth parts of the second differences of X Y Z.

The values of P, Q, R for April 17 are enclosed in brackets to indicate that their values have been inferred, and not calculated: the values of x, y, z for May 3 are enclosed in brackets to indicate that they depend upon the preceding values of P, Q, R.

Table exhibiting Details of Calculation of provisional Orbit of J. VIII.

	Jan. 28.	Feb. 13.	Feb. 29.	Mar. 16.	Apr. 1.	Apr. 17.	May 3.
x	+ ·1113290	+ ·0974232	+ ·0827044	+ ·0672506	+ ·0511598	+ ·0345560	(+ ·0175977)
y	− ·1570880	− ·1624748	− ·1664110	− ·1686981	− ·1691203	− ·1674444	(− ·1634199)
z	− ·0531070	− ·0447827	− ·0359985	− ·0268150	− ·0173081	− ·0075732	(+ ·0022697)
x'	+ 3·2999	+ 3·3949	+ 3·4882	+ 3·5797	+ 3·6694	+ 3·7572	+ 3·8432
y'	− 4·1496	− 4·0791	− 4·0065	− 3·9318	− 3·8551	− 3·7764	− 3·6960
z'	− 0·0587	− 0·0611	− 0·0635	− 0·0658	− 0·0681	+ 0·0704	− 0·0727
X_1	− ·0010107	− ·0009553	− ·0008821	− ·0007863	− ·0006613	− ·0004987	
Y_1	+ ·0014262	+ ·0015931	+ ·0017749	+ ·0019724	+ ·0021861	+ ·0024163	
Z_1	+ ·0004821	+ ·0004391	+ ·0003839	+ ·0003135	+ ·0002237	+ ·0001093	
X_2	+ ·0018109	+ ·0018614	+ ·0019080	+ ·0019507	+ ·0019889	+ ·0020229	
Y_2	− ·0022675	− ·0022109	− ·0021515	− ·020899	− ·0020262	− ·0019611	
Z_2	− ·0000032	− ·0000092	− ·0000154	− ·0000217	− ·0000279	− ·0000341	
X_3	− ·0016771	− ·0017204	− ·0017625	− ·0018036	− ·0018435	− ·0018823	
Y_3	+ ·0021090	+ ·0020670	+ ·0020243	+ 0019809	− ·0019368	+ ·0018920	
Z_3	+ ·0000298	+ ·0000310	+ ·0000321	+ ·0000332	+ ·0000342	+ ·0000353	
X	− ·0008769	− ·0008143	− ·0007366	− ·0006392	− ·0005159	− ·0003581	
Y	+ ·0012677	+ ·0014492	+ ·0016477	+ ·0018634	+ ·0020967	+ ·0023472	
Z	+ ·0005087	+ ·0004609	+ ·0004006	+ ·0003250	+ ·0002300	+ ·0001105	
P	...	+ ·0000013	+ ·0000016	+ ·0000022	+ ·0000029	(+ ·0000036)	
Q	...	+ ·0000014	+ ·0000014	+ ·0000015	+ ·0000014	(+ ·0000014)	
R	...	− ·0000010	− ·0000013	− ·0000016	− ·0000020	(− ·0000025)	

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From the co-ordinates x, y, z of the preceding table, the tabular differences of geocentric right ascension and declination between the satellite and Jupiter were obtained. We exhibit also the interpolation for every fourth day.

Table for Paris Mean Noon, giving excess of Satellite's tabular R.A. and Decl. over that of Jupiter.

Date.	R.A.	Decl.	Date.	R.A.	Decl.
1908.	s	″		s	″
Jan. 28	−7·29	−2588·7	Mar. 20	+112·71	−1419·6
Feb. 1	+1·19	2511·9	24	122·83	1332·4
5	9·72	2430·4	28	132·99	1247·7
9	18·34	2345·0	Apr. 1	143·15	1165·7
13	27·07	2256·4	5	153·27	1086·7
17	35·96	2165·3	9	163·32	1010·7
21	45·01	2072·2	13	173·25	937·8
25	54·22	1977·8	17	183·02	868·0
29	63·60	1883·0	21	192·60	801·4
Mar. 4	73·15	1788·3	25	201·95	737·8
8	82·85	1694·0	29	211·03	677·3
12	92·69	1602·3	May 3	+219·81	−619·7
16	+102·65	−1509·3			

The next table gives the dates of the observations reduced to Paris time with the light times subtracted; the place of observation, and the observed geocentric distances in R.A. and declination of the satellite from Jupiter; also the excess of the corresponding calculated co-ordinates over the observed.

Comparison of Provisional Orbit with Observation.

Date.	Hour.	Place.	Observed R.A. J. VIII—♃.	Calculated minus Observed.	Observed Decl. J. VIII—♃.	Calculated minus Observed.
1908.	h		s	s	″	″
Jan. 27	12·2	Greenwich	− 8·29	−0·04	−2597·3	−0·6
Feb. 1	11·4	,,	+ 2·20	0·00	2501·1	−1·3
3	10·0	,,	6·37	−0·04	2464·2	+1·0
22	10·5	,,	48·38	−0·07	2038·1	−0·3
23	8·1	,,	50·31	+0·07	2015·7	−1·4
24	12·0	,,	53·15	−0·09	1989·2	−0·4
27	10·5	,,	59·94	−0·02	1919·7	−0·4
28	11·0	,,	62·34	−0·02	1895·2	−0·7
Mar. 3	9·3	Heidelberg	71·72	−0·03	1804·6	+1·8
8	19·9	Lick	84·72	+0·16	1674·4	−0·4

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Comparison of Provisional Orbit with Observation—continued.

Date.	Hour.	Place.	Observed R.A. J. VIII—♃.	Calculated minus Observed.	Observed Decl. J. VIII—♃.	Calculated minus Observed.
1908.	h		s	s	″	″
Mar. 23	7·1	Heidelberg	121·12	−0·07	1347·1	−0·5
27	8·4	Greenwich	131·35	−0·01	1259·9	−1·4
31	8·8	,,	141·55	−0·01	1178·1	−0·4
Apr. 1	16·3	Lick	145·00	−0·13	1153·7	+1·6
3	9·3	Greenwich	149·33	−0·14	1120·2	+2·0
24	9·2	,,	201·03	−0·52	1745·7	−1·7
29	16·3	Lick	+213·08	−0·55	− 669·9	+2·6

It will be noticed that the accordance is exceedingly close, considering that this is a first approximation to the orbit, and that nothing whatever has been done in the way of modifying the orbit to produce a better accordance.

It remains to modify the orbit, and then to carry it forward to the end of the year, to assist in picking up the satellite at the next opposition of Jupiter.

For general interest, from the first three lines of the first table of this paper the following jovicentric longitudes, latitudes, and distances are computed, also L the longitude in the orbit reckoned from the ascending node on the ecliptic.

1908.	Long.	Lat.	L.	Distance.
	°	°	°	
Jan. 28	305·32	−15·42	328·42	·1997
Feb. 13	300·95	−13·31	333·18	·1946
29	296·43	−10·96	338·17	·1893
Mar. 16	291·74	− 8·40	343·45	·1836
Apr. 1	287·83	− 5·59	349·09	·1775
17	281·66	− 2·54	355·09	·1711
May 3	276·15	+ 0·79	1·53	·1644

The instantaneous orbit is inclined 31°·14 to the ecliptic, and 29°·84 to Jupiter's orbit, and the instantaneous eccentricity has a value about $\frac{1}{3}$; the mean distance is about 0·1702, and the period about two years and two months.

The longitude of ascending node is 277°·46.

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Observations of Jupiter's Sixth, Seventh, and Eighth Satellites from photographs taken with the 30-inch Reflector at the Royal Observatory, Greenwich, in 1907–8.

(*Communicated by the Astronomer Royal.*)

The sixth, seventh, and eighth satellites of Jupiter have been under observation during the recent opposition of Jupiter with the 30-inch reflector, photographs having been obtained whenever possible. In all 38 photographs of Satellite VI, 21 photographs of Satellite VII, and 13 photographs of Satellite VIII have been secured between 1907 November 13 and 1908 April 24.

The positions of the satellites have been measured on the photographs taken with the reflector with reference to faint comparison stars (of eleventh or twelfth magnitude), symmetrically distributed about the satellites. The positions of these faint comparison stars were then measured relatively to the reference stars (of eighth to ninth magnitude) on photographs (with two exposures of 20^m or 30^m) taken with the Astrographic 13-inch refractor. The field, sensibly free from distortion, being much larger with this telescope than with the reflector, from 24 to 39 reference stars were available on each plate. As Jupiter moved slowly, it was possible to make one reference plate serve for several photographs, which were each referred to it. The constants were determined in the usual manner, all the stars on the plate given in the Astronomische Gesellschaft catalogues being used for this purpose. Right ascensions and declinations of the satellites were then determined, and compared with the corrected tabular places of Jupiter after applying the corrections

$$\text{in R.A.} + 0^s{\cdot}110, \quad \text{and} \quad \text{Dec.} + 0''{\cdot}72$$

as determined below.

The G.M.T. is the arithmetical mean of the times of beginning and end of exposure.

SATELLITE VI.

Date and G.M.T.				Exposure.	Apparent R.A.			Apparent Dec.			Sat. VI—Jupiter.			
											R.A.		Dec.	
1907.	d	h	m	min.	h	m	s	°	′	″	m	s	′	″
Nov.	13	13	47·8	60	8	59	54·16	+17	27	6·2	−3	10·90	+3	40·9
,,	15	14	57·9	70	9	0	11·88	17	24	54·3	3	18·89	+2	52·2
Dec.	5	14	15·7	40	9	0	38·63	17	15	14·9	4	14·48	−5	24·2
,,	5	16	1·7	140	9	0	38·31	17	15	15·7	4	14·52	5	25·6
,,	10	15	7·3	66	9	0	2·87	17	16	17·9	4	21·25	7	34·7
,,	13	13	14·8	13	8	59	34·29	17	17	34·1	4	23·89	8	49·6
,,	14	14	48·9	120	8	59	22·64	17	18	7·9	4	24·44	9	17·8